

Modeling of Aircraft Unsteady Aerodynamic Characteristics

Part 3 - Parameters Estimated From Flight Data

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SUMMARY

A nonlinear least squares algorithm for aircraft parameter estimation from flight data was developed. The postulated model for the analysis represented longitudinal, short period motion of an aircraft. The corresponding aerodynamic model equations included indicial functions (unsteady terms) and conventional stability and control derivatives. The indicial functions were modelled as simple exponential functions. The estimation procedure was applied in five examples. Four of the examples used simulated and flight data from small amplitude maneuvers of the F-18 HARV and X-31A aircraft. In the fifth example a rapid, large amplitude maneuver of the X-31 drop model was analyzed. From data analysis of small amplitude maneuvers it was found that the model with conventional stability and control derivatives was adequate. Also, parameter estimation from a rapid, large amplitude maneuver did not reveal any noticeable presence of unsteady aerodynamics.

SYMBOLS

a, a_α, a_q	parameters in indicial functions
a_x, a_z	longitudinal and vertical accelerations, g units
B_α, B_q	parameters defined in table I
b_I	parameter in indicial function, $1/sec$
C_L, C_m, C_Z	lift, pitching-moment and vertical-force coefficients
C_α, C_q	parameters defined in table I
$C_{L_\alpha}(t), C_{Z_\alpha}(t), C_{m_\alpha}(t)$	indicial functions
$Cov(\theta)$	parameter covariance matrix
c, c_α, c_q	parameters in indicial functions
c_α^*, c_q^*	parameters in indicial functions defined in eq. (23)
\bar{c}	mean aerodynamic chord, m
$F(t), F_{L_\alpha}(t)$	deficiency functions
g	acceleration due to gravity, m/sec^2
$h - h_0$	location of aircraft center of gravity from the aerodynamic center of the wing expressed in \bar{c}
H	sensitivity matrix
I_x, I_y, I_z	moments of inertia about longitudinal, lateral, and vertical body axes, $kg \cdot m^2$
I_{xz}	product of inertia, $kg \cdot m^2$
$J(\theta)$	cost function
ℓ	characteristic length, m
ℓ_t	tail arm, m
m	mass, kg

N	number of data points
n_p	number of parameters
p, q, r	roll rate, pitch rate, and yaw rate, <i>rad/sec</i> or <i>deg/sec</i>
\bar{q}	dynamic pressure, $\rho V^2/2$, Pa
R^2	coefficient of determination
S	wing area, m^2
s	standard error
T	time lag, ℓ/V , sec
t	time, <i>sec</i>
V	airspeed, <i>m/sec</i>
x_α	state variable in eq. (10)
y	dependent variable
z	measured dependent variable
α	angle of attack, <i>rad</i> or <i>deg</i>
δ	control surface deflection, <i>rad</i> or <i>deg</i>
δ_c, δ_v	canard and thrust vectoring vane deflection, <i>rad</i> or <i>deg</i>
Θ	vector of unknown parameters
θ	unknown parameter
ε	a) measurement error b) error due to approximation (Appendix A)
ρ	air density, kg/m^3
τ	time delay, <i>sec</i>
ω	angular frequency, <i>rad/sec</i>

Superscripts:

- derivative with respect to time
- ^ estimate

Subscripts:

- t tail
- w wing
- $a.c.$ aerodynamic center

Matrix exponents:

- T transpose matrix
- -1 inverse matrix

Derivatives of aerodynamic coefficients C_a where the index $a = L, Z$, or m

$$C_{a_q} = \frac{\partial C_a}{\partial \frac{q\bar{c}}{2V}}, \quad C_{a_\alpha} = \frac{\partial C_a}{\partial \alpha}, \quad C_{a_\delta} = \frac{\partial C_a}{\partial \delta}, \quad C_{a_{\dot{\alpha}}} = \frac{\partial C_a}{\partial \frac{\dot{\alpha}\bar{c}}{2V}}$$

derivatives $C_{a_\alpha}^*$ and $C_{a_\delta}^*$ defined in eq. (22)

derivatives $L_{\alpha, \dot{\alpha}, q, \delta}$ and $M_{\alpha, \dot{\alpha}, q, \delta}$ defined in table I.

INTRODUCTION

Possible effects of unsteady aerodynamics on aircraft motion were investigated as early as in the forties and fifties (see reference 1 and 2). So far, however, there have been only a few attempts to analyze flight data with the intent to estimate parameters characterizing unsteady aerodynamics. In reference 3, a procedure for estimation of unsteady aerodynamic terms was proposed and applied to longitudinal data of a scaled model. In this procedure, the conventional stability and control derivatives were estimated first. Then, the resulting residuals were used for estimation of unsteady terms. Reference 4 addressed the formulation of indicial functions in the equations of motion and the identifiability of these functions. Examples were given using simulated data and then flight data of a drop model from the post-stall and spin region. It was found that the unsteady part of the model was not identifiable for the responses below the angle of attack of 40° . In reference 5 a simple vortex system was used to incorporate the unsteady aerodynamics into the longitudinal equations of motion. Neither simulated nor flight data of a general aviation aircraft undergoing small amplitude maneuvers at low angles of attack showed any significant difference in parameter estimates using either the model with or without unsteady terms. Finally, in reference 6 and 7 an unsteady aerodynamic model was determined from longitudinal, large amplitude maneuvers. In this case the modeling was based on internal state variables rather than indicial functions. From the investigation, however, it was not clear how the model with time-invariant parameters would explain the measured response.

The present report is the third part of the series devoted to the modeling of unsteady aerodynamics in the equations of motion and to estimation of aerodynamic parameters from experimental data. In the first part of this series (reference 8), linear models for aerodynamic forces and moments were formulated. In the second part (reference 9), developed models were used in the analysis of wind tunnel data from forced oscillation test. The purpose of this report is to postulate an aircraft model with a simple form for the unsteady aerodynamics, investigate the effect of unsteady terms on the motion of an aircraft and estimate the aerodynamic parameters in the postulated model from flight data. Because the report is

considered as a preliminary study into the problem, all examples presented, except one, are limited to the longitudinal, short period motion with linear aerodynamic equations. Only the last example investigates the adequacy of a model without unsteady terms for a longitudinal, large amplitude maneuver. The report starts with the formulation of model equations followed by the development of a parameter estimation algorithm based on the least square procedure. Then, the algorithm is applied to simulated and flight data. The results obtained are discussed and concluding remarks drawn.

EQUATIONS OF MOTION

As an example of aircraft motion with unsteady aerodynamic terms the short period longitudinal motion will be studied. The development of these equations is similar to that in reference 8. The short period longitudinal motion can be described as

$$\begin{aligned}\dot{\alpha} &= q - \frac{\rho V S}{2m} C_L(\alpha(t), q(t), \delta(t)) \\ \dot{q} &= \frac{\rho V^2 S \bar{c}}{2I_y} C_m(\alpha(t), q(t), \delta(t))\end{aligned}\tag{1}$$

It is further assumed that the aerodynamic model equations can be formulated as

$$\begin{aligned}C_L(t) &= \int_0^t C_{L_\alpha}(t-\tau) \frac{d}{d\tau} \alpha(\tau) d\tau + \frac{\ell}{V} C_{L_q}(\infty) q(t) + C_{L_\delta}(\infty) \delta(t) \\ C_m(t) &= \int_0^t C_{m_\alpha}(t-\tau) \frac{d}{d\tau} \alpha(\tau) d\tau + \frac{\ell}{V} C_{m_q}(\infty) q(t) + C_{m_\delta}(\infty) \delta(t)\end{aligned}\tag{2}$$

where $C_{L_\alpha}(t)$ and $C_{m_\alpha}(t)$ are indicial functions representing the response in C_L and C_m to a unit step in α , and the remaining coefficients are the conventional stability and control derivatives. In a more general case they might be replaced by indicial functions.

The indicial functions in equation (2) achieve steady state values $C_{L_\alpha}(\infty)$ and $C_{m_\alpha}(\infty)$ respectively. Because of this property, the indicial function $C_{L_\alpha}(t)$ can be expressed as

$$C_{L_\alpha}(t) = C_{L_\alpha}(\infty) - F_{L_\alpha}(t)\tag{3}$$

where $F_{L_\alpha}(t)$ is called the deficiency function (ref. 2). A similar relationship holds for $C_{m_\alpha}(t)$.

The most difficult part of modeling aerodynamics in eq. (1) is the postulation of expressions for indicial functions. If the model is to be used in parameter estimation from experimental data, then it should be parsimonious. This means that the model must fit the data well with the smallest number of parameters. The problem of formulating aircraft indicial functions was addressed in references 1, 2, 5, 10 and 11. For this

study, the approach of references 10 and 11 was used. Reference 10 extends the two-dimensional potential theory of an airfoil in nonuniform motion to a wing with finite aspect ratio and elliptic spanwise loading. Further extension to tapered, swept wings in incompressible flow is covered in reference 11. It is shown that in all cases the indicial function $C_{L_\alpha}(t)$ can be formulated as

$$\begin{aligned} C_{L_\alpha}(t) &= a(1 - e^{-b_1 t}) + c \\ &= C_{L_\alpha}(\infty) - ae^{-b_1 t} \end{aligned} \tag{4}$$

which was also the form used in ref. 9 for the analysis of wind tunnel data.

The indicial function for a wing-tail combination, however, becomes more complicated. The indicial function $C_{L_\alpha}(t)$ must be considered to represent the combined effect between the wing and tail. In linear theory the resulting indicial function is equal to the sum of the components (see ref. 2):

- (1) the indicial function of the wing alone, the tail being at zero angle of attack;
- (2) the indicial function of the tail alone, the wing being at zero angle of attack;
- (3) the response in lift of the tail to a step change in angle of attack of the wing, the tail being at zero angle of attack;
- (4) the response in lift of the wing to a step change in angle of attack of the tail, the wing being at zero angle of attack.

The last two components express the interference effects.

Reference 2 presents a detailed qualitative analysis for supersonic speed. References 1 and 5 cover the subsonic case by concentrating on the effect of the wing wake on the lift of the tail and neglecting the influence of the tail on the wing. Examples from reference 10 and 1 are reproduced in figures 1a and 1b. In these figures the indicial function of an elliptic wing with aspect ratio 6 and the indicial function of a tail with aspect ratio 3 are plotted against the nondimensional time, $Vt/(\bar{c}/2)$. As seen on figure 1b the lift on the tail becomes infinite when the leading edge of the tail reaches the position of the starting vortex. After that, the lift decreases and, with increasing time, approaches a steady value. Although the indicial lift

shows an infinite value, the convolution with the angle of attack results in finite lift at all points. The dotted line in figure 1b is an approximation to $C_{L_{t\alpha}}(t)$ representing the development of the lift on the tail by a single lag function. However, the lag is greater than that indicated by the tail length.

The resulting indicial functions for the lift and pitching moment of the whole aircraft can be obtained from the expressions

$$C_L(t) = C_{L_w}(t) + \frac{S_t}{S} C_{L_t}(t) \quad (5)$$

$$C_m(t) = C_{m_{a.c}} + (h - h_0) C_{L_w}(t) - \frac{\ell_t S_t}{\bar{c} S} C_{L_t}(t) \quad (6)$$

In more detailed analysis, C_{L_w} should be replaced by the lift of a wing-body combination. Using the results in figure 1a and 1b, the indicial function $C_{m_\alpha}(t)$ and its approximation were computed from equation (6) for $C_{m_{a.c}} = 0$, $h - h_0 = 0.1$ and the tail volume equal to 0.6. Both functions are plotted in figure 1c where the dotted line indicates the approximation.

For the following analysis the indicial functions are approximated by rather simple forms as

$$\begin{aligned} C_{L_\alpha}(t) &= C_{L_\alpha}(\infty) - a_\alpha e^{-b_l t} \\ C_{m_\alpha}(t) &= C_{m_\alpha}(\infty) - a_q e^{-b_l t} \end{aligned} \quad (7)$$

which means that the pitching-moment indicial function may be suitable for a flying wing and perhaps for an aircraft with a short tail length. Substituting eq. (7) into eq. (2) and integrating by parts results in aerodynamic equations

$$\begin{aligned} C_L(t) &= c_\alpha \alpha(t) + a_\alpha b_l \int_0^t e^{-b_l \tau} \alpha(t - \tau) d\tau + \frac{\ell}{V} C_{L_q} q(t) + C_{L_\delta} \delta(t) \\ C_m(t) &= c_q \alpha(t) + a_q b_l \int_0^t e^{-b_l \tau} \alpha(t - \tau) d\tau + \frac{\ell}{V} C_{m_q} q(t) + C_{m_\delta} \delta(t) \end{aligned} \quad (8)$$

where the simplified notation for aerodynamic derivatives was used. Combining eq. (1) and (8), and introducing dimensional parameters, the equations of motion can be written as

$$\begin{aligned}\dot{\alpha} &= -C_\alpha \alpha(t) - B_\alpha \int_0^t e^{-b_I \tau} \alpha(t-\tau) d\tau + (1-L_q)q - L_\delta \delta \\ \dot{q} &= C_q \alpha(t) + B_q \int_0^t e^{-b_I \tau} \alpha(t-\tau) d\tau + M_q q + M_\delta \delta\end{aligned}\quad (9)$$

where the parameters in these equation are defined in table 1. Introducing a new state variable

$$x_\alpha = \int_0^t e^{-b_I \tau} \alpha(t-\tau) d\tau$$

and the corresponding state equation for this variable

$$\dot{x}_\alpha = \alpha - b_I x_\alpha$$

equations (9) can be rewritten in the state-space form as

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{x}_\alpha \end{bmatrix} = \begin{bmatrix} -C_\alpha & 1-L_q & -B_\alpha \\ C_q & M_q & B_q \\ 1 & 0 & -b_I \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ x_\alpha \end{bmatrix} + \begin{bmatrix} -L_\delta \\ M_\delta \\ 0 \end{bmatrix} \delta \quad (10)$$

If no unsteady aerodynamics are considered, eq. (2) are simplified as

$$\begin{aligned}C_L(t) &= C_{L_\alpha} \alpha(t) + \frac{\ell}{V} \left(C_{L_{\dot{\alpha}}} \dot{\alpha}(t) + C_{L_q} q(t) \right) + C_{L_\delta} \delta(t) \\ C_m(t) &= C_{m_\alpha} \alpha(t) + \frac{\ell}{V} \left(C_{m_{\dot{\alpha}}} \dot{\alpha}(t) + C_{m_q} q(t) \right) + C_{m_\delta} \delta(t)\end{aligned}\quad (11)$$

and the longitudinal short period equations will take the form

$$\begin{bmatrix} 1+L_{\dot{\alpha}} & 0 \\ -M_{\dot{\alpha}} & 1 \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -L_\alpha & 1-L_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -L_\delta \\ M_\delta \end{bmatrix} \delta \quad (12)$$

where the parameters $L_{\dot{\alpha}}$, $M_{\dot{\alpha}}$, L_α and M_α are also defined in table 1.

Equations (11) represent an approximation to the more general form given by eq. (2). The difference between those two formulations can be interpreted as an error due to omitting the unsteady aerodynamic terms. The dependence of this error on the aircraft geometry and motion variables is investigated in reference 4 and Appendix A. It was found that for a

is investigated in reference 4 and Appendix A. It was found that for a harmonic motion with given frequency ω , the relative error is equal or less than the reduced frequency of the motion, i.e.

$$\text{relative error} \leq \frac{\omega \ell}{V}$$

where ℓ is a characteristic length. This result indicates that the error will increase with increased frequency of motion and characteristic length and decrease with the airspeed. During transient longitudinal motion, the prevailing frequency is close to the natural frequency of the aircraft. The characteristic length will depend on the physics of the flow which creates unsteady effects. For unsteady lift on the wing $\ell = \bar{c}$, for the interaction of the wing and the tail ℓ is equal to the tail length, $\ell = \ell_t$. When fuselage vortices are interacting with the tail, ℓ will be equal to the fuselage length. However, the airspeed during the maneuver will have the main effect on reduced frequency. Therefore the unsteady effects will be more pronounced at low airspeed which means at high angles of attack.

PARAMETER ESTIMATION ALGORITHM

A regression method for estimation of aerodynamic parameters from measured input/output time histories was selected because of its simplicity. Considering aerodynamic model equations (8), the regression equation for each of the model equations can be written as

$$z(i) = \theta_1 \alpha(i) + \theta_2 \theta_3 \int_0^t e^{-\theta_3 \tau} \alpha(t_i - \tau) d\tau + \theta_4 \frac{\ell}{V} q(i) + \theta_5 \delta(i) + \varepsilon(i) \quad (13)$$

$$i = 1, 2, \dots, N$$

In this equation θ_j are the unknown parameters, $z(i)$ is the measured independent variable, $\varepsilon(i)$ is the measurement error and N is the number of data points. For the pitching-moment coefficient

$$\theta_1 = c_q, \quad \theta_3 = b_l, \quad \theta_5 = C_{m_\delta}$$

$$\theta_2 = a_q, \quad \theta_4 = C_{m_q}$$

and $z(i) = C_m(i)$. The parameter estimates are obtained by minimizing the cost function

$$J = \sum_{i=1}^N \varepsilon_i^2 = \sum_{i=1}^N [z(i) - y(i)]^2 \quad (14)$$

where $y(i)$ is the computed independent variable

Because regression equation (13) is nonlinear in the parameters the minimum of the cost functions is computed from the linearized form

$$J = \sum_{i=1}^N \left[z(i) - y_0(i) - \sum_{j=1}^{n_p} \frac{\partial y(i)}{\partial \theta_j} \Delta \theta_j \right]^2 \quad (15)$$

where $\partial y / \partial \theta_j$ are the sensitivities

$$\begin{aligned} \frac{\partial y}{\partial \theta_1} &= \alpha, & \frac{\partial y}{\partial \theta_2} &= b_1 \int_0^t e^{-b_1 \tau} \alpha(t - \tau) d\tau, \\ \frac{\partial y}{\partial \theta_3} &= a_q \int_0^t (1 - b_1 \tau) e^{-b_1 \tau} \alpha(t - \tau) d\tau, \\ \frac{\partial y}{\partial \theta_4} &= \frac{\ell}{V} q, & \frac{\partial y}{\partial \theta_5} &= \delta, \end{aligned}$$

The sensitivities and the independent variable y_0 are computed for nominal values of the parameters. Minimization of (15) leads to an iterative procedure where the estimates after the r th iteration are equal to

$$\hat{\Theta}_{r+1} = \hat{\Theta}_r + \Delta \hat{\Theta}_r \quad (16)$$

The parameter updates $\Delta \hat{\Theta}_j$ are obtained as

$$\Delta \hat{\Theta} = \left[\sum_{i=1}^N H_i^T H_i \right]^{-1} \sum_{i=1}^N H_i^T (z(i) - y_0(i)) \quad (17)$$

where H is the sensitivity matrix

$$H = \left\{ \frac{\partial y}{\partial \theta_j} \right\}, \quad j = 1, 2, \dots, n_p$$

Assuming that α, q , and δ are measured without errors and that ϵ forms a random, white sequence, the parameter covariance matrix can be estimated as

$$\text{Cov}(\hat{\Theta}) = s^2 \left[\sum_{i=1}^N H_i^T H_i \right]^{-1} \quad (18)$$

where

$$s^2 = \frac{J(\hat{\Theta})}{N - n_p}$$

For the aerodynamic equation expressed as

$$C_m(t) = C_{m_\alpha} \alpha(t) - a_q \int_0^t e^{-b_1 \tau} \frac{d}{d\tau} \alpha(t - \tau) d\tau + \frac{\ell}{V} C_{m_q} q(t) + C_{m_\delta} \delta(t) \quad (19)$$

the corresponding regression equation is

$$z(i) = \theta_1 \alpha(i) - \theta_2 \int_0^t e^{-\theta_3 \tau} \frac{d}{d\tau} \alpha(t_i - \tau) d\tau + \theta_4 \frac{\ell}{V} q(i) + \theta_5 \delta(i) \quad (20)$$

and the sensitivities and are

$$\begin{aligned} \frac{\partial y}{\partial \theta_2} &= - \int_0^t e^{-\theta_3 \tau} \frac{d}{d\tau} \alpha(t - \tau) d\tau \\ \frac{\partial y}{\partial \theta_3} &= a_q \int_0^t \tau e^{-\theta_3 \tau} \frac{d}{d\tau} \alpha(t - \tau) d\tau \end{aligned}$$

If the parameter, θ_3 , in equation (13) or (20) is assumed to be known, the estimation of the remaining parameters is reduced to an ordinary least squares method.

A different estimation scheme based on the least-squares principle was introduced in reference 12. It transforms state equation (1) and equation (2) into the complex domain. Then the modulating function technique of reference 13 is applied to those equations in order to obtain a computationally suitable form of the regression equations. These equations are linear in the parameters. The estimates of aerodynamic parameters in (2), however, require a solution of nonlinear algebraic equations. These equations could be rather complicated even for a small number of unknown parameters. For that reason the algorithm of reference 12 was not used in this report.

ANALYSIS OF SIMULATED DATA

Two examples of simulated data were generated to explore the effects of unsteady aerodynamics on system output variables and aerodynamic parameters estimated from the input/output data. Both examples are of aircraft performing small-amplitude longitudinal maneuvers from trim conditions at 30° angle of attack. The model used to generate the data was that given by equations (10) and (12). In addition to the computed output variables, α and q , the time histories of the lift and pitching moment coefficients with unsteady aerodynamic terms were also computed from eq. (8). Three different approaches were used for estimation of unknown parameters. In the first approach the aerodynamic model with only

conventional stability and control derivatives (see eg. (11)) was considered. In the remaining two cases, the correct model given by equation (8) was applied with the parameter b_I either fixed or estimated. Thus the first and second approach resulted in a simple linear regression. For the third approach, the estimation algorithm described by equations (15) to (18) was applied.

Advanced Fighter Aircraft

Aircraft characteristics and flight conditions for an advanced fighter aircraft are summarized in table II. These characteristics are close to those of the F-18 High-Angle-of-Attack Research Vehicle (HARV) as reported in reference 14. The value of parameters c , a and b_I in the indicial functions are included in table III and IV. The computed time histories of the angle of attack, pitch rate and two coefficients C_L and C_m are plotted in figures 2 and 3. Figure 2 also includes the input variable $\delta(t)$. Both the response variables and aerodynamic coefficients show only small differences between their values from equations with and without unsteady aerodynamics.

For parameter estimation, measurement noise was added to the coefficient C_L and C_m computed with unsteady terms. The noise was white and Gaussian with zero mean and variance $\sigma^2 = 6.9 \times 10^{-3}$ for C_L and 1.55×10^{-4} for C_m . The results from three difference cases are given in tables III and IV. Included are parameter values and their standard errors, coefficients of determinations, R^2 , which define the amount of information in the data explained by the model, and the standard errors of the coefficients, $s(C_L)$ or $s(C_m)$, which represent fit errors. The estimation algorithm which includes b_I as an unknown parameter did not converge where the data with the measurement noise were used. Convergence was obtained only with the reduced amount of noise. In the given example the noise was reduced to 1/20 of the original measurement noise in C_L and to 1/3 of the original value for C_m .

The model with conventional stability and control derivatives fitted the data well and the estimated parameters were close to their true values. One should remember that, in this case, the estimate of C_{m_q} represents the

sum of C_{m_q} and $C_{m_{\dot{\alpha}}}$ with the true value of -12.5. The remaining estimates of $C_{m_{\alpha}}$ and $C_{m_{\delta}}$ are also affected by terms containing $C_{m_{\dot{\alpha}}}$. Similarly, the parameter estimates of $C_{L_{\alpha}}$, C_{L_q} and $C_{L_{\delta}}$ contain a small contribution due to $C_{L_{\dot{\alpha}}}$. When the unsteady model was used with the parameter b_1 fixed, no noticeable improvements in R^2 , $s(C_L)$ or $s(C_m)$ were observed. There was, however, an increase in the standard errors of parameters accompanied by high correlation between parameters c_{α} and C_{L_q} , a_{α} and c_{α} , and a_{α} and C_{L_q} as can be seen in table V. The same applies for the correlation of parameters in the pitching-moment equation. By adding the parameter b_1 to the unknown parameters, the correlation of parameters become even worse than in the previous case. As indicated by table VI, all three parameters c_{α} , a_{α} and b_1 , or c_q , a_q and b_1 are highly correlated. Improvement in the accuracy of the estimated parameters is a result of decreased measurement noise variance.

Experimental Aircraft

The aircraft characteristics and flight conditions for this example are presented in Table VII. The characteristics were selected close to those of the X-31A aircraft. This aircraft has three longitudinal controllers, the tailing edge flaps, canard and thrust vectoring. In the model developed, it was assumed that the control system generates the canard deflection, δ_c , proportional to α , and thrust deflection, δ_{tv} , proportional to tailing edge flaps deflection, δ . The lift coefficient expressed with conventional stability and control derivatives has the form

$$\begin{aligned} C_L(t) &= C_{L_{\alpha}} \alpha(t) + \frac{\ell}{V} \left(C_{L_{\dot{\alpha}}} \dot{\alpha}(t) + C_{L_q} \dot{q}(t) \right) + C_{L_{\delta}} \delta(t) + C_{L_{\delta_c}} \delta_c(t) \\ &\quad + C_{L_{\delta_{tv}}} \delta_{tv}(t) \\ &= C_{L_{\alpha}}^* \alpha(t) + \frac{\ell}{V} \left(C_{L_{\dot{\alpha}}} \dot{\alpha}(t) + C_{L_q} \dot{q}(t) \right) + C_{L_{\delta}}^* \delta(t) \end{aligned} \quad (21)$$

where

$$\begin{aligned} C_{L_{\alpha}}^* &= C_{L_{\alpha}} + k_{\alpha} C_{L_{\delta_c}} \\ C_{L_{\delta}}^* &= C_{L_{\delta}} + k_{\delta} C_{L_{\delta_{tv}}} \end{aligned} \quad (22)$$

Similar expressions hold for $C_m(t)$. The aerodynamic equations with unsteady aerodynamics were obtained from equation (8) with c_α and c_q replaced by

$$\begin{aligned} c_\alpha^* &= C_{L_\alpha}^* - a_\alpha \\ c_q^* &= C_{m_\alpha}^* - a_q \end{aligned} \quad (23)$$

The values of parameters a_α , a_q and b_1 , were estimate from wind tunnel oscillatory data as described in reference 9. The values of these parameters are in the second column of tables VIII and IX.

As follows from figures 4 and 5, the effect of unsteady aerodynamics is now more pronounced than in the previous example. For parameter estimation, the time histories $C_L(t)$ and $C_m(t)$ computed from modified equations (8) were again corrupted by measurement noise with the variance of 4.0×10^{-3} for C_L and 1.19×10^{-4} for C_m . The increased significance of unsteady aerodynamics was reflected in the accuracy of the estimated parameters and in the value of R^2 . When compared with estimates related to equation (21). The estimation with the correct model and fixed parameter b_1 moved the parameter mean values closer to the true values, but there was an increase in parameter standard errors. There was some improvement in R^2 in the lift equation, whereas the coefficient of determination in the pitching-moment equation changed only slightly. Finally, where all parameters were estimated their accuracy decreased and high correlation between parameters of the indicial functions appeared. These results are all given in tables VIII and IX.

ANALYSIS OF FLIGHT DATA

Several longitudinal, short period maneuvers of the F-18 HARV and X-31A aircraft were analyzed. The purpose of this analysis was to estimate parameters in the model with and without unsteady aerodynamics and to decide whether a simple model with stability and control derivatives is adequate for each of these maneuvers. Reported are, however, only two examples, one for each aircraft. Conclusions from this set of data are similar to those from the remaining maneuvers. The third example, based on data of the X-31 drop model, was added to demonstrate the adequacy of a

model without unsteady terms for a longitudinal, high amplitude maneuver.

Prior to parameter estimation the measured data were checked for their compatibility and estimated bias errors (constant offset and/or scale factors) were removed from the data. Then, the aerodynamic coefficients of the vertical force and pitching moment were computed as

$$C_Z = \frac{mg}{\bar{q}S} a_z$$

$$C_m = \frac{I_y}{\bar{q}S\bar{c}} \left[\dot{q} - \frac{I_z - I_x}{I_y} pr - \frac{I_{xz}}{I_y} (r^2 - p^2) \right]$$

where the angular acceleration in pitch was obtained by numerical differentiation of measured pitching velocity. For parameter estimation the coefficient C_Z rather than C_L was used. It was assumed that the accuracy of calculated C_Z would be higher than that of C_L because that lift coefficient is computed from time histories of two accelerations, angle of attack and thrust.

F-18 HARV

The measured time histories of longitudinal variables from a transient maneuver about $\alpha \approx 28^\circ$ are given in figure 6. This maneuver was excited by an optimal input implemented by the pilot (see reference 15) and was performed without thrust vectoring. The measured coefficients C_Z and C_m are presented in figure 7. For parameter estimation, a model without unsteady aerodynamics and a stepwise regression of reference 15 were applied first. An adequate model for C_Z contained only two regressors, α and q , and resulted in coefficient of determination, $R^2 = 97.53$. For the coefficient C_m an adequate model included three regressors, α , q and δ , and the coefficient of determination was equal to 95.63. The predicted coefficients C_Z and C_m from this analysis are compared with those measured in figure 7 where the corresponding residuals are also plotted. Both the values of R^2 and the residuals indicate that a simple model can explain a substantial part of variation in the data. Regardless of these findings, parameters in the unsteady model given by equation (8) were estimated for b_1 fixed on several values ranging from 0.5

to 1.5. No improvements in R^2 and in the residuals were observed. In addition, the parameter accuracy was degraded and strong multiple correlations among the parameters α and c , and C_{Z_q} or C_{m_q} were observed.

X-31A Aircraft

The longitudinal maneuver presented was excited at $\alpha \approx 68^\circ$ by three controllers, trailing edge flaps, canard and thrust vectoring. The measured time histories of three output and three control variables are plotted in figure 8. The measured coefficients C_Z and C_m are given in figure 9. Parameters were estimated in the same way as in the previous example. The predicted coefficients from estimated stability and control derivatives are compared with measured values in figure 9. The corresponding residuals are also included. The values of R^2 for C_Z and C_m were equal to 95.15 and 96.89 respectively. The residuals in C_Z include some low frequency components caused probably by external disturbances. Parameter estimation in the model with unsteady terms did not increase the R^2 value and did not significantly change the residuals. Because the residuals in C_m resemble a random, white sequence, and because of the high value of R^2 , no further parameter estimations in the pitching-moment equation with different models was attempted.

X-31 Drop Model

The X-31 drop model is a 27% scale model of the actual aircraft. A brief description of the model and testing procedure can be found in reference 17. The measured data containing time histories of three output and two input variables are presented in figure 10. The data indicate that in the maneuver, the angle of attack was changed from 22° to 75° and back to 20° within 3.5 sec. The maximum pitch rate reached was $80^\circ/\text{sec}$ which corresponds to $42^\circ/\text{sec}$ for the full scale aircraft. The measured aerodynamic coefficients C_Z , C_L and C_m are shown in figure 11. The lift coefficient was obtained from measured data as

$$C_L = \frac{mg}{\bar{q}S} (a_x \sin \alpha - a_z \cos \alpha)$$

Possible nonlinearities in the aerodynamic model equations without unsteady aerodynamics were modeled by polynomial splines, see reference 18. These equations were formed as

$$C_a = C_a(\alpha) \Big|_{\substack{q=\delta=0 \\ \delta_c=-40^\circ}} + C_{a_q}(\alpha) \frac{q\bar{c}}{2V} + C_{a_\delta}(\alpha)\delta + C_{a_{\delta_c}}(\alpha)\Delta\delta_c$$

$$a = Z, L, \text{ or } m$$

where

$C_a(\alpha), \dots, C_{a_\delta}(\alpha)$ are polynomial splines of the first or second order,

$$\Delta\delta_c = \delta_c + 0.698$$

The stepwise regression was used to determine an adequate model for each coefficient from postulated spline terms and estimated parameter in these models. The predicted aerodynamic coefficients and residuals are plotted in figure 11. The values of the coefficients of determination were equal to 99.81, 98.79 and 99.81 for C_Z , C_L and C_m respectively. Both the plots and R^2 values indicate that the models with time-invariant parameters explain almost all variation in measured data.

DISCUSSION OF RESULTS

Models for the longitudinal, short period motion used in this report contained two indicial functions, $C_{L_\alpha}(t)$ or $C_{Z_\alpha}(t)$, and $C_{m_\alpha}(t)$. The remaining terms in the aerodynamic model equations were represented by conventional stability and control derivatives. The analytical expression for the indicial function was selected in the form of a single exponential function with three parameters, a , c , and b_1 , see eq. (4). Values of the first two parameters are constrained by the condition that their sum is equal to the derivative C_{Z_α} , C_{L_α} , or C_{m_α} . A priori values for parameters a and b_1 can be obtained from theory or wind tunnel experiment. As indicated earlier, the form of indicial functions used in simulation and flight data analysis can approximate unsteady aerodynamics of a wing alone. The use of the same form for the indicial function $C_{m_\alpha}(t)$ of a wing-tail combination is questionable. It can be substantiated only as a first approximation before more rigorous analysis is attempted.

Examples with simulated and measured data revealed that it might be difficult or even impossible to detect unsteady effects in data from

longitudinal, small amplitude maneuvers for two main reasons: these effects can be small enough so that they cannot be distinguished from the measurement noise and/or unknown modeling errors; even if there are no modeling errors and low measurement noise in the data, the identifiability of parameters in indicial functions is in doubt. Further, the examples showed that a model with conventional stability and control derivatives could be adequate. The model with unsteady aerodynamics did not improve the fit to the data. In addition, its use resulted in parameter estimates with low accuracy and high correlations. From the limited theoretical analysis and small amount of examples it can be concluded that, in general, parameter estimation might detect unsteady aerodynamic effects in measured data from small amplitude maneuvers if the transient motion is initiated at low speed and contains high frequency components, the characteristic length is large, and the aircraft has high gradients $dC_L/d\alpha$ and $dC_m/d\alpha$. It is also important that the measured data were corrected for bias errors and have a high signal-to-noise ratio.

The last example of this report dealt with a rapid, large amplitude maneuver. It was expected that the effect of unsteady aerodynamics would be detected. The model with a small number of time-invariant parameters, however, explained almost all the variation in the data. This indicates that even that type of maneuver can not guarantee a noticeable presence of unsteady aerodynamics.

CONCLUDING REMARKS

A nonlinear least squares algorithm for aircraft parameter estimation from flight data was developed. The postulated model for the analysis represented longitudinal, short period motion of an aircraft. The corresponding aerodynamic model equations included indicial functions (unsteady terms) and conventional stability and control derivatives. In formulating the analytical form for the indicial functions, two conflicting requirements had to be taken into account: parameter estimation requires a simple model with a small number of parameters in order to improve or insure their identifiability; indicial functions should be good approximations to complex physical phenomena associated with unsteady and separated flow. The model proposed in this study was formed as a

simple exponential function which can approximate unsteady aerodynamics of a wing alone, but is questionable for the indicial function in the pitching-moment equation of a wing-tail combination.

The estimation procedure was applied in four examples to simulated and flight data from small amplitude maneuvers of the F-18 High-Angle-of-Attack Research Vehicle (HARV) and X-31 A aircraft. Then, in the fifth example a rapid, large amplitude maneuver of the X-31 drop model was analyzed. From brief theoretical study and examples, the following conclusions can be drawn:

1. A possibility for detecting unsteady effects in measured data will be enhanced if the transient motion is initiated at low speed, the response contains high frequency components, aircraft characteristic length is large, and the angle-of-attack gradient of the lift and pitching moment are high;
2. from data analysis of small amplitude maneuvers it was found that the model with conventional stability and control derivatives was adequate;
3. the model with unsteady aerodynamics did not improve the fit to the data over the simple model without unsteady terms. More complex models resulted in parameter estimates with low accuracy and high correlations;
4. parameter estimation from a rapid, large amplitude maneuver also did not reveal any noticeable presence of unsteady aerodynamics.

Despite the negative results from this study, more extensive research is needed to determine how important the modeling of unsteady aerodynamics is on aircraft motion.

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APPENDIX A

Approximation of Unsteady Aerodynamic Terms

As an example, the pitching-moment coefficient is formulated with and without the unsteady term as

$$C_m = C_{m_0} + C_{m_\alpha} \alpha - \int_0^T F(t - \tau) \dot{\alpha}(\tau) d\tau + \frac{\ell}{V} C_{m_q} q \quad (\text{A1})$$

and

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + \frac{\ell}{V} (C_{m_{\dot{\alpha}}} \dot{\alpha} + C_{m_q} q) \quad (\text{A2})$$

where the time T in the upper limit of the integral can be viewed as a delay after which the indicial function approaches its steady value. Thus T is proportional to the time the flow needs to travel a characteristic distance ℓ , which means that $T = \ell/V$.

The difference between eq. (A1) and (A2) represents an error due to approximation

$$\varepsilon' = \int_0^T F(t - \tau) \dot{\alpha}(\tau) d\tau + \frac{\ell}{V} C_{m_{\dot{\alpha}}} \dot{\alpha} \quad (\text{A3})$$

It has been shown in ref. 7 that, under some simplifying assumptions, the counterpart to $C_{m_{\dot{\alpha}}}$ is proportional to the area of the deficiency function, that is

$$C_{m_{\dot{\alpha}}} = -\frac{V}{\ell} \int_0^T F(\tau) d\tau \quad (\text{A4})$$

substituting (A4) into (A3)

$$\begin{aligned} \varepsilon' &= \int_0^T F(t - \tau) \dot{\alpha}(\tau) d\tau - \dot{\alpha} \int_0^T F(\tau) d\tau \\ &= \int_0^T F(\tau) [\dot{\alpha}(t - \tau) - \dot{\alpha}(t)] d\tau \end{aligned} \quad (\text{A5})$$

Since

$$|\alpha(t - \tau) - \alpha(\tau)| \leq \tau |\dot{\alpha}_{\max}(t)|$$

then also

$$|\dot{\alpha}(t - \tau) - \dot{\alpha}(\tau)| \leq \tau |\ddot{\alpha}_{\max}(t)|$$

Using the last inequality the error can be further expressed as

$$\begin{aligned}\varepsilon' &\leq \int_0^T |F(\tau)| |\dot{\alpha}(t-\tau) - \dot{\alpha}(\tau)| d\tau \\ &\leq \int_0^T |F(\tau)| \tau |\ddot{\alpha}_{\max}(t)| d\tau \leq T |\ddot{\alpha}_{\max}(t)| \int_0^T |F(\tau)| d\tau\end{aligned}\quad (\text{A6})$$

The maximum value of the integral in eq.(A1) is

$$|\dot{\alpha}_{\max}(t)| \int_0^T |F(\tau)| d\tau$$

which is used as a normalizing factor in the expression for relative error

$$\varepsilon \leq \frac{\varepsilon'}{|\dot{\alpha}_{\max}(t)| \int_0^T |F(\tau)| d\tau} \leq \frac{|\ddot{\alpha}_{\max}(t)|}{|\dot{\alpha}_{\max}(t)|} T \quad (\text{A7})$$

For a simple harmonic motion, the relative error takes the form

$$\varepsilon \leq \omega T = \frac{\omega \ell}{V} \quad (\text{A8})$$

where ω is the angular frequency

Table I. - Definition of parameters in equations.

$$\begin{aligned}
 L_\alpha &= \frac{\rho S V}{2m} C_{L_\alpha} & M_\alpha &= \frac{\rho V^2 S \bar{c}}{2I_Y} C_{m_\alpha} \\
 L_{\dot{\alpha}} &= \frac{\rho S \bar{c}}{4m} C_{L_{\dot{\alpha}}} & M_{\dot{\alpha}} &= \frac{\rho V S \bar{c}^2}{4I_Y} C_{m_{\dot{\alpha}}} \\
 L_q &= \frac{\rho S \bar{c}}{4m} C_{L_q} & M_q &= \frac{\rho V S \bar{c}^2}{4I_Y} C_{m_q} \\
 L_\delta &= \frac{\rho S V}{2m} C_{L_\delta} & M_\delta &= \frac{\rho V^2 S \bar{c}}{2I_Y} C_{m_\delta} \\
 C_\alpha &= \frac{\rho S V}{2m} c_\alpha & C_q &= \frac{\rho V^2 S \bar{c}}{2I_Y} c_q \\
 B_\alpha &= \frac{\rho S V}{2m} a_\alpha b_1 & B_q &= \frac{\rho V^2 S \bar{c}}{2I_Y} a_q b_1
 \end{aligned}$$

Table II. - Characteristics of an advanced fighter aircraft and flight conditions.

$$\begin{aligned}
 \bar{c} &= 3.51 \text{ m} & C_{L_\alpha} &= 2.7 \\
 S &= 37.16 \text{ m}^2 & C_{L_{\dot{\alpha}}} &= 2.5 \\
 m &= 15000 \text{ kg} & C_{L_q} &= 36. \\
 I_Y &= 170000 \text{ kg} \cdot \text{m}^2 & C_{L_\delta} &= 0.83 \\
 \rho &= 0.56 \text{ kg} / \text{m}^3 & C_{m_\alpha} &= -0.18 \\
 V &= 90 \text{ m} / \text{sec} & C_{m_{\dot{\alpha}}} &= -2.5 \\
 & & C_{m_q} &= -10. \\
 & & C_{m_\delta} &= -0.88
 \end{aligned}$$

**Table III. Estimated parameters from simulated data.
Advanced fighter aircraft.**

Parameter	True value	Estimates (a)		
		No unsteady effects	b_I known	b_I estimated (c)
c_α	2.75	—	2.4 (0.23)	2.75 (0.063)
a_α	-0.05	—	0.4 (0.25)	-0.04 (0.060)
b_I	1.0	—	1.0 —	1. (1.3)
C_{L_q}	36.0	36. (3.7)	49. (9.6)	36. (1.2)
C_{L_δ}	0.83	0.7 (0.10)	0.8 (0.13)	0.829 (0.0091)
C_{L_α}	2.70	2.71 (0.043)	2.8 (b) (0.25)	2.71 (b) (0.062)
$R^2(\%)$	84.35	83.57	83.61	99.95
$s(C_L)$	0.0828	0.0829	0.0828	0.0000172

^a Numbers in parenthesis are standard errors.

^b Computed from $C_{L_\alpha} = a_\alpha + c_\alpha$.

^c Measurement noise reduced to 1/20 of its previous value.

**Table IV. Estimated parameters from simulated data.
Advanced fighter aircraft.**

Parameter	True value	Estimates (a)		
		No unsteady effects	b_I known	b_I estimated (c)
c_q	-0.23	—	-0.17 (0.034)	-0.23 (0.064)
a_q	0.05	—	-0.02 (0.038)	0.04 (0.060)
b_I	1.0	—	1.0 —	1. (1.3)
C_{m_q}	-10.0	-11.5 (0.56)	-12. (1.4)	-10. (1.2)
C_{m_δ}	-0.88	-0.88 (0.016)	-0.88 (0.019)	-0.879 (0.0091)
C_{m_α}	-0.18	-0.186 (0.0064)	-0.19 (b) (0.037)	-0.2 (b) (0.12)
$R^2(\%)$	85.45	83.00	83.00	97.82
$s(C_m)$	0.00124	0.00124	0.0125	0.0000172

a Numbers in parenthesis are standard errors.

b Computed from $C_{m_\alpha} = a_q + c_q$.

c Measurement noise reduced to 1/3 of its previous value.

Table V. Correlation matrix of parameters ($b_I = 1.0$)

c_α	C_{L_q}	C_{L_δ}	a_α
1.000	-0.929	-0.586	-0.982
	1.000	0.689	0.922
		1.000	0.565
			1.000

Table VI. Correlation matrix of parameters

c_α	a_α	b_I	C_{L_q}	C_{L_δ}
1.000	-0.999	-0.961	-0.970	-0.778
	1.000	0.952	0.975	0.783
		1.000	0.879	0.671
			1.000	0.849
				1.000

Table VII. - Characteristics of an experimental aircraft and flight conditions.

$\bar{c} = 3.76 \text{ m}$	$C_{L_\alpha}^* = 1.12$
$S = 21.02 \text{ m}^2$	$C_{L_{\dot{\alpha}}} = 66.8$
$m = 6700 \text{ kg}$	$C_{L_q} = 1.0$
$I_Y = 46792 \text{ kg} \cdot \text{m}^2$	$C_{L_\delta}^* = 1.22$
$\bar{q} = 2211 \text{ Pa}$	$C_{m_\alpha}^* = -0.142$
$V = 101 \text{ m/sec}$	$C_{m_{\dot{\alpha}}} = -9.08$
	$C_{m_q} = -1.29$
	$C_{m_\delta}^* = -0.71$

**Table VIII. Estimated parameters from simulated data.
Experimental aircraft.**

Parameter	True value	Estimates (a)		
		No unsteady effects	b_1 known	b_1 estimated
c_α^*	2.55	—	2.4 (0.11)	2.8 (0.54)
a_α	-1.43	—	-1.2 (0.14)	-1.5 (0.53)
b_1	1.15	—	1.15 —	1.4 (0.34)
C_{L_q}	1.0	44. (2.0)	8. (4.7)	1. (11.)
$C_{L_\delta}^*$	1.22	1.72 (0.085)	1.25 (0.099)	1.2 (0.14)
$C_{L_\alpha}^*$	1.12	1.50 (0.025)	1.2 (b) (0.21)	1. (b) (1.1)
$R^2(\%)$	84.88	82.03	84.88	83.25
$s(C_L)$	0.0632	0.0654	0.0632	0.0632

^a Numbers in parenthesis are standard errors.

^b Computed from $C_{L_\alpha}^* = a_\alpha + c_\alpha^*$.

Table IX. Estimated parameters from simulated data.
Experimental aircraft.

Parameter	True value	Estimates (a)		
		No unsteady effects	b_I known	b_I estimated
c_q^*	-0.49	—	-0.42 (0.052)	-0.9 (0.67)
a_q	0.35	—	0.028 (0.056)	0.8 (0.66)
b_I	2.07	—	2.07 —	3. (1.3)
C_{m_q}	-1.29	-9.4 (0.34)	-3. (1.3)	2. (6.7)
$C_{m_\delta}^*$	-0.71	-0.79 (0.014)	-0.72 (0.020)	-0.69 (0.038)
$C_{m_\alpha}^*$	-0.14	-0.166 (0.0042)	-0.1 (b) (0.11)	-0.1 (b) (1.1)
$R^2(\%)$	85.31	82.59	83.00	83.03
$s(C_m)$	0.0109	0.0110	0.0109	0.0109

a Numbers in parenthesis are standard errors.

b Computed from $C_{m_\alpha}^* = a_q + c_q^*$.

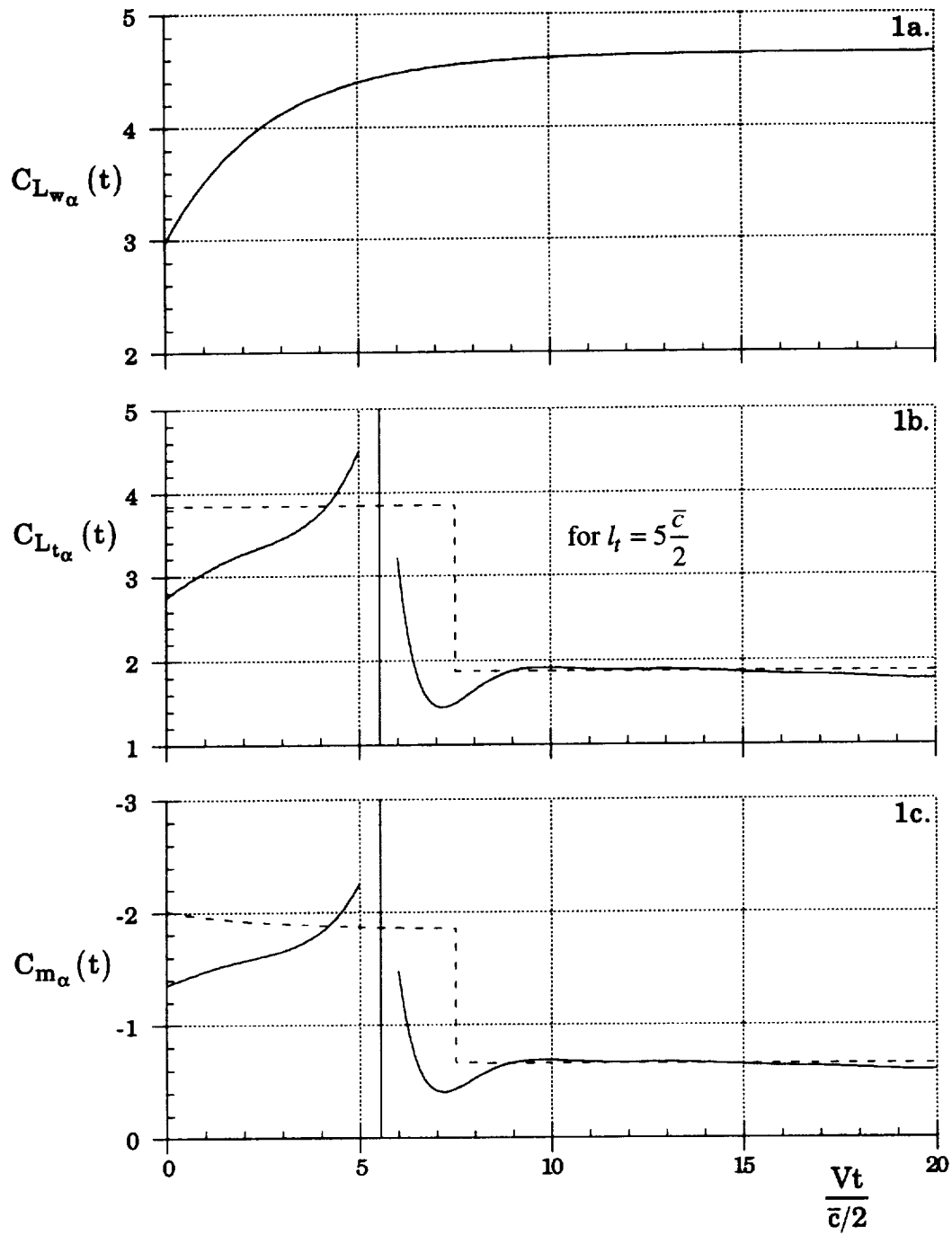


Figure 1. Indicial function for wing (1a.), tail (1b.), and wing / tail combination (1c.).

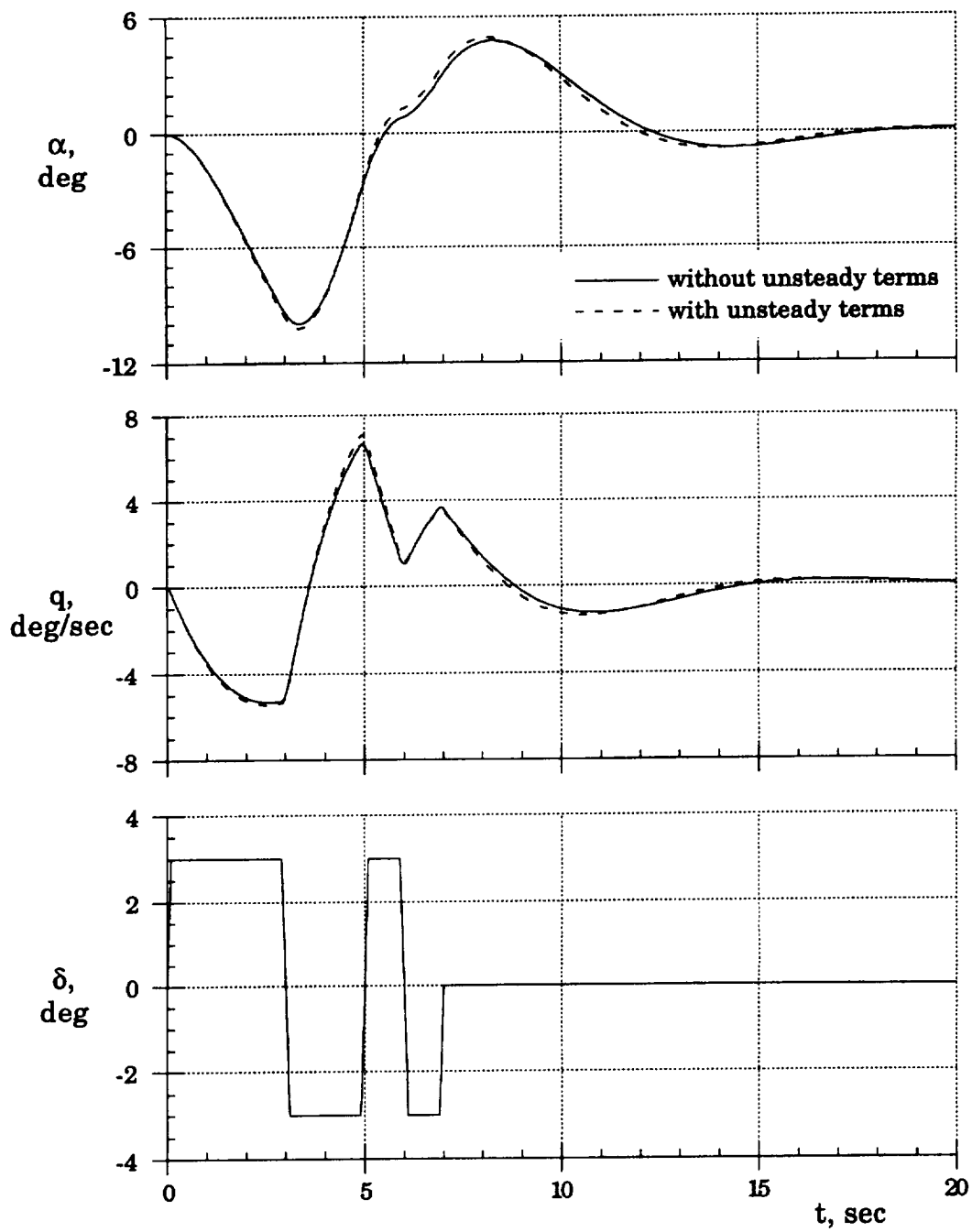


Figure 2. Computed time histories with and without unsteady aerodynamic terms. Advanced fighter aircraft.

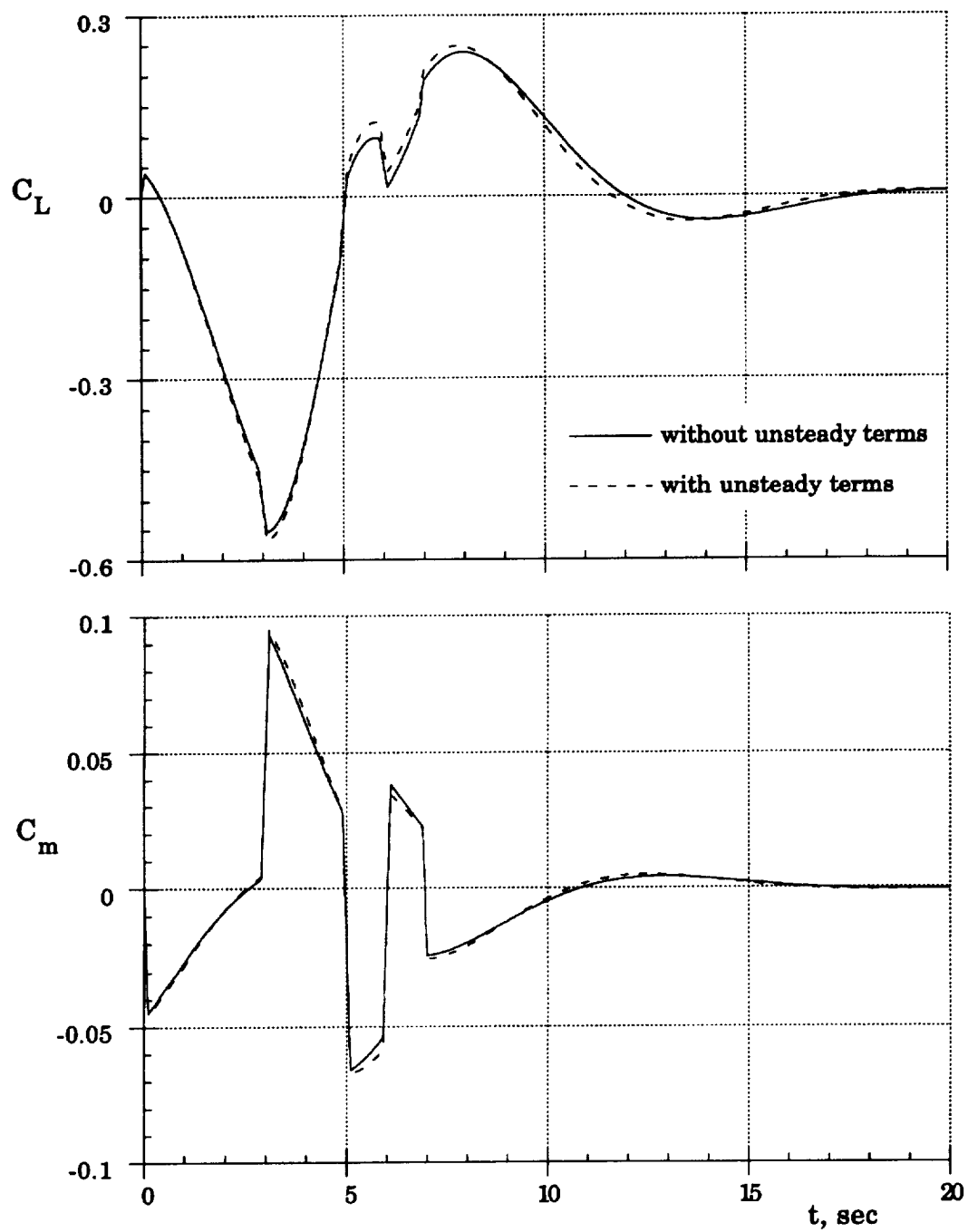


Figure 3. Computed time histories of aerodynamic coefficients with and without unsteady aerodynamic terms. Advanced fighter aircraft.

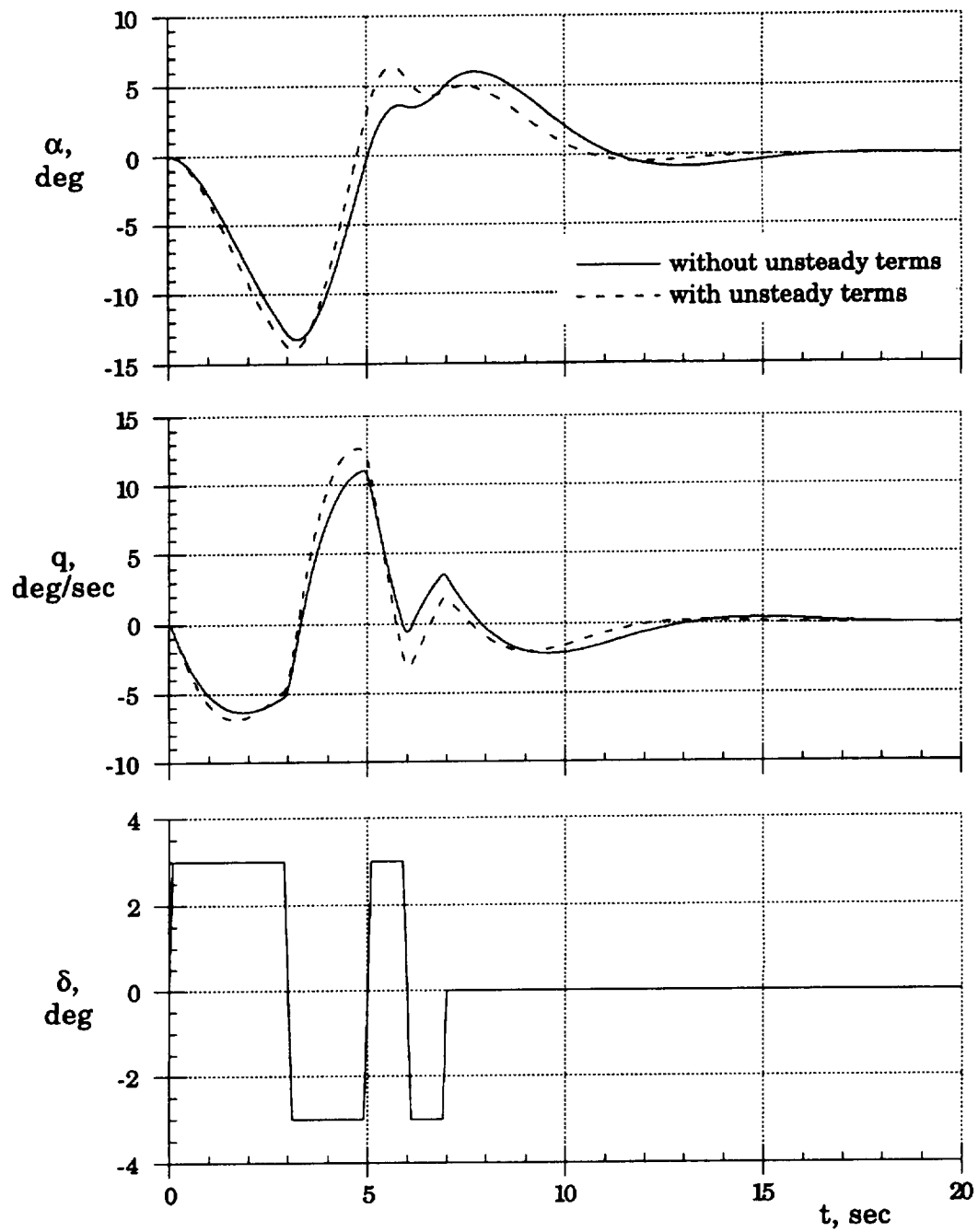


Figure 4. Computed time histories with and without unsteady aerodynamic terms. Experimental aircraft.

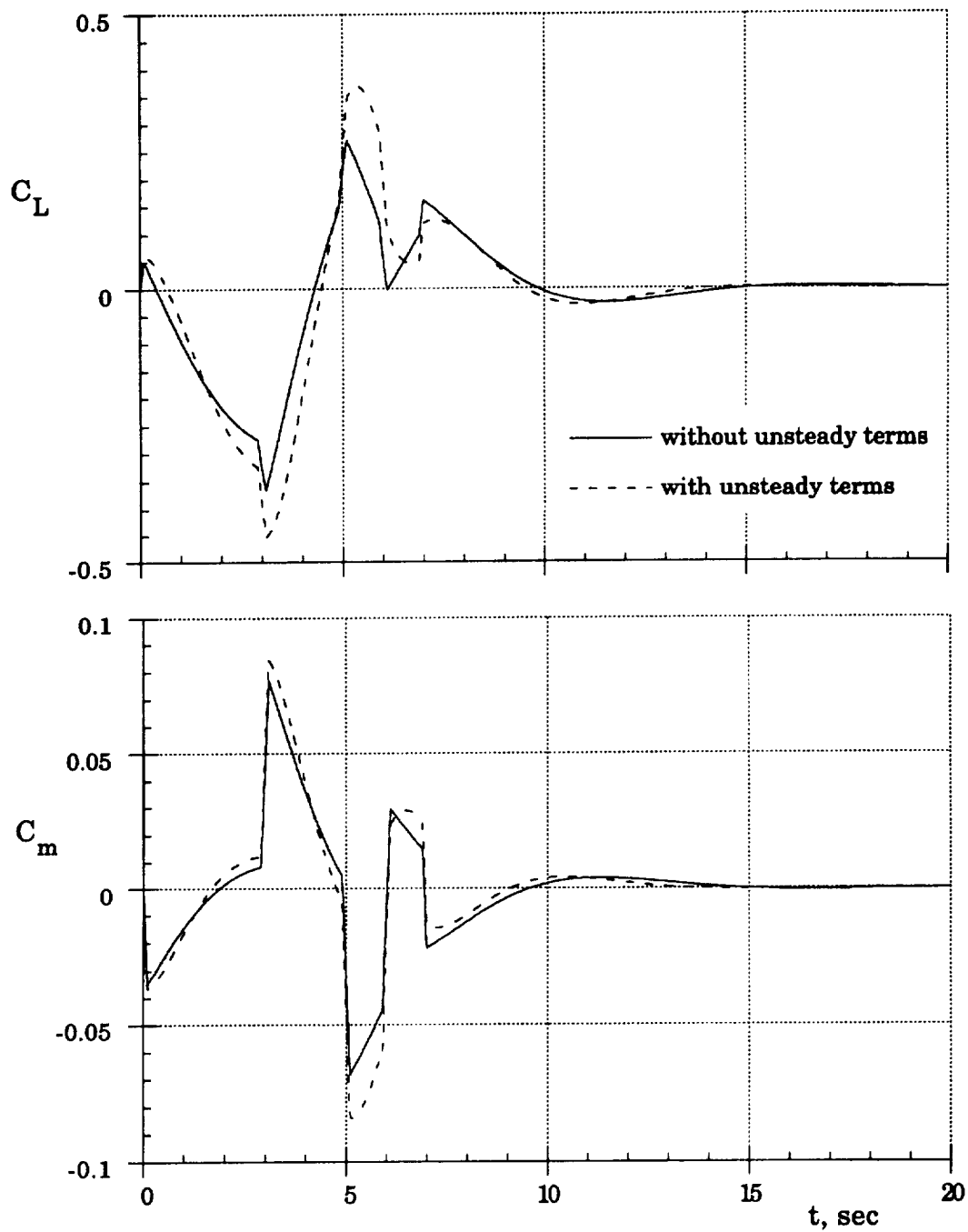


Figure 5. Computed time histories of aerodynamic coefficients with and without unsteady aerodynamic terms. Experimental aircraft.

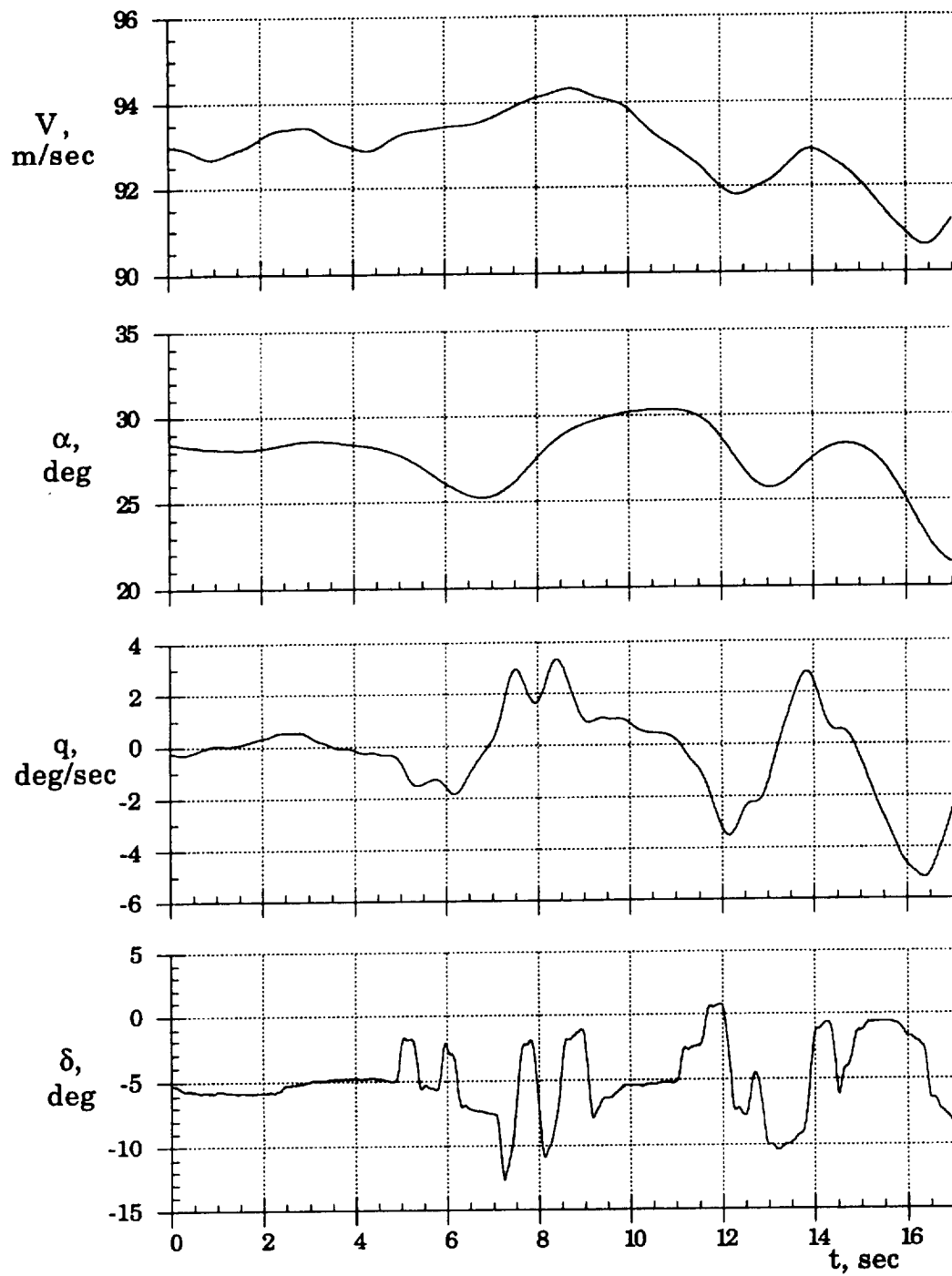


Figure 6. Time histories of measured longitudinal variables.
F-18 HARV.

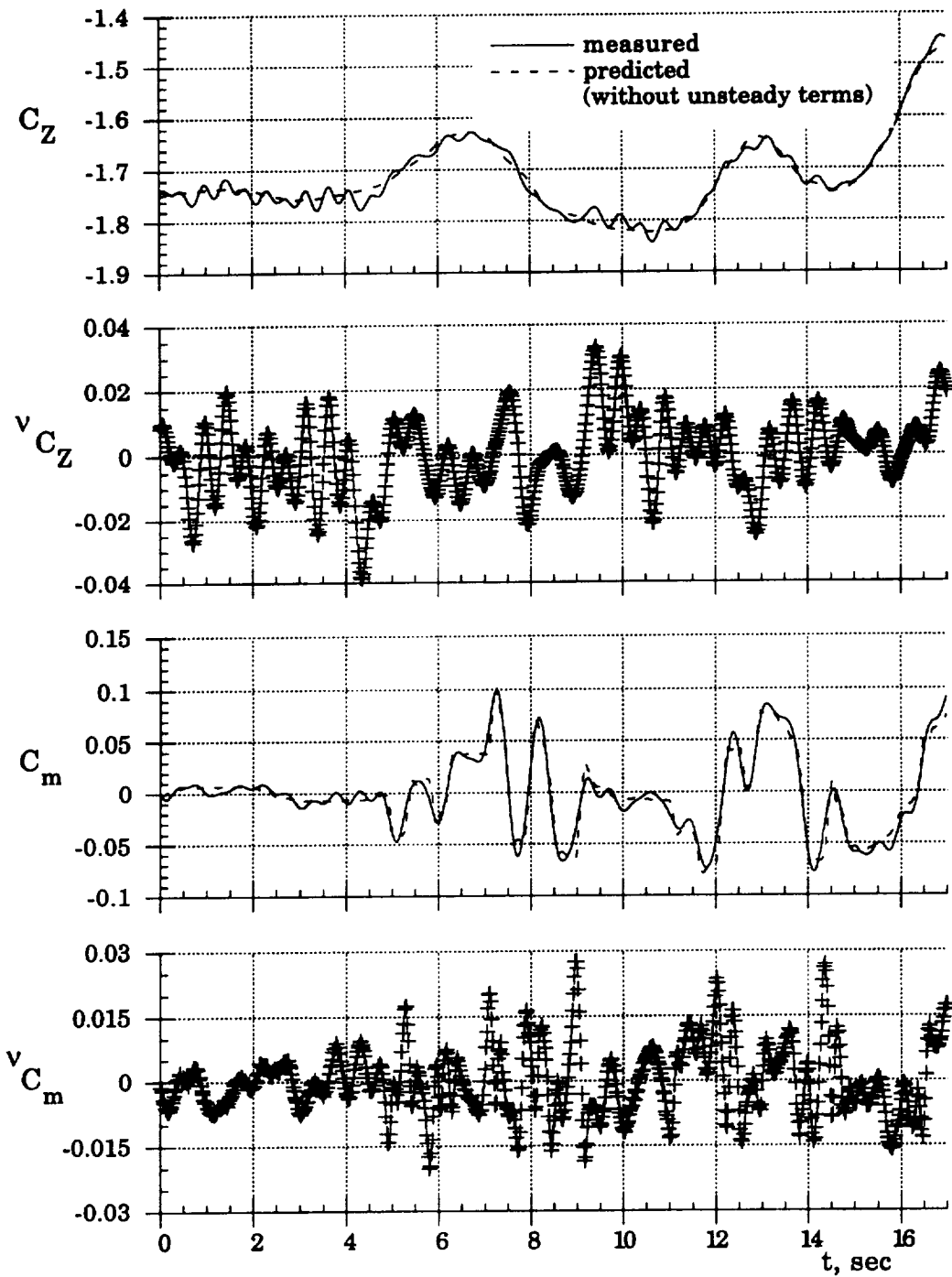


Figure 7. Time histories of measured and predicted aerodynamic coefficients and residuals. F-18 HARV.

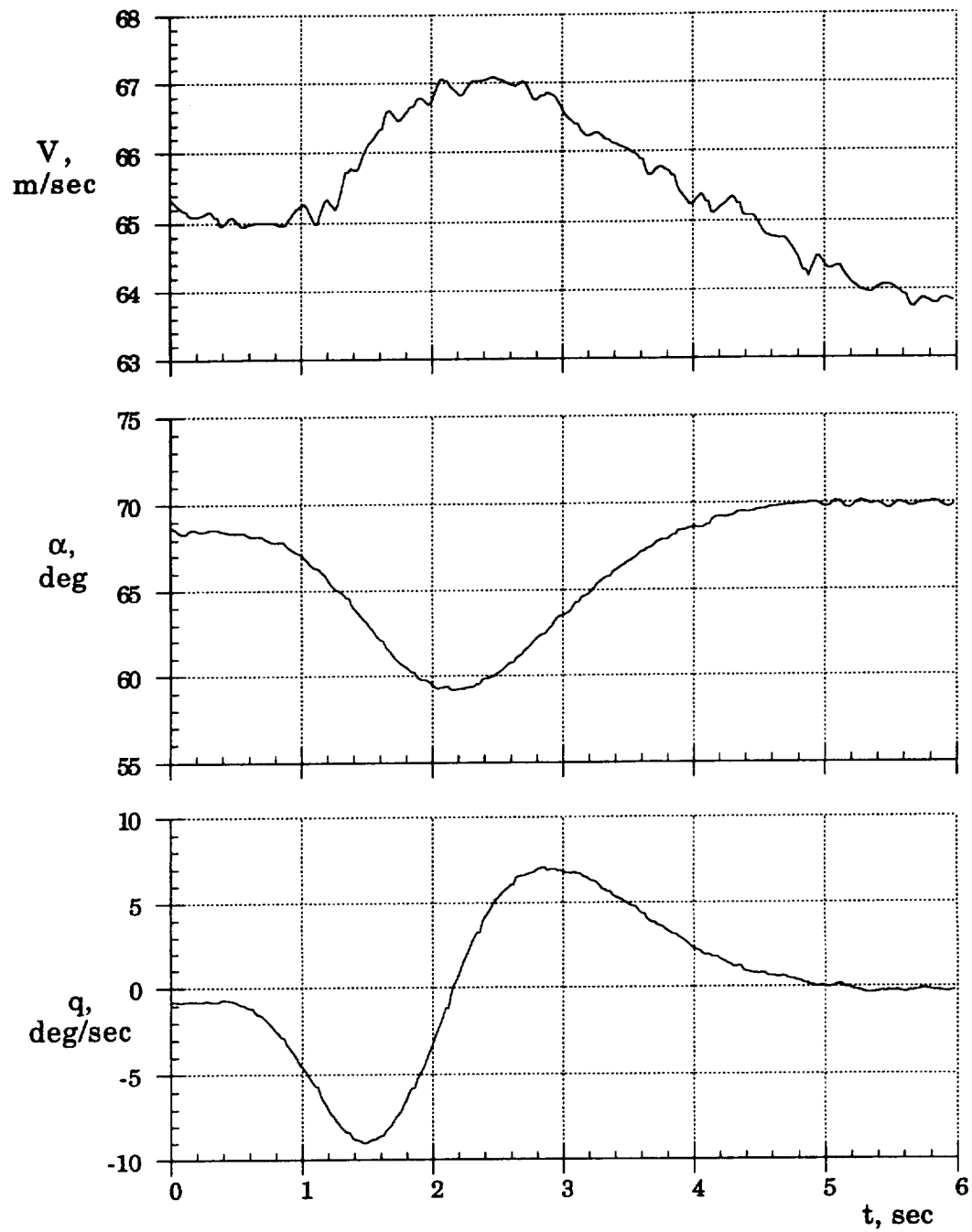


Figure 8. Time histories of measured longitudinal variables. X-31A aircraft.

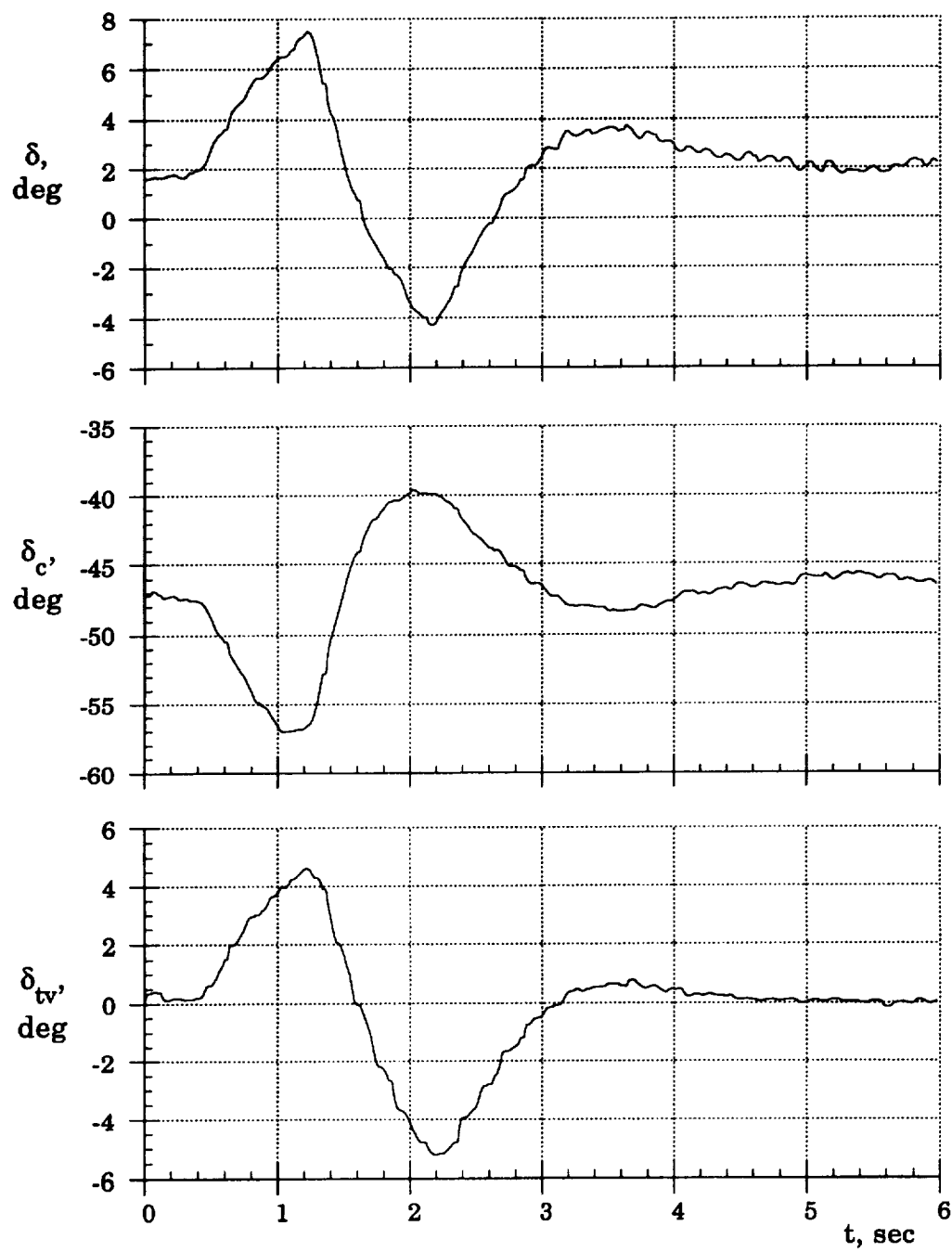


Figure 8. Concluded.

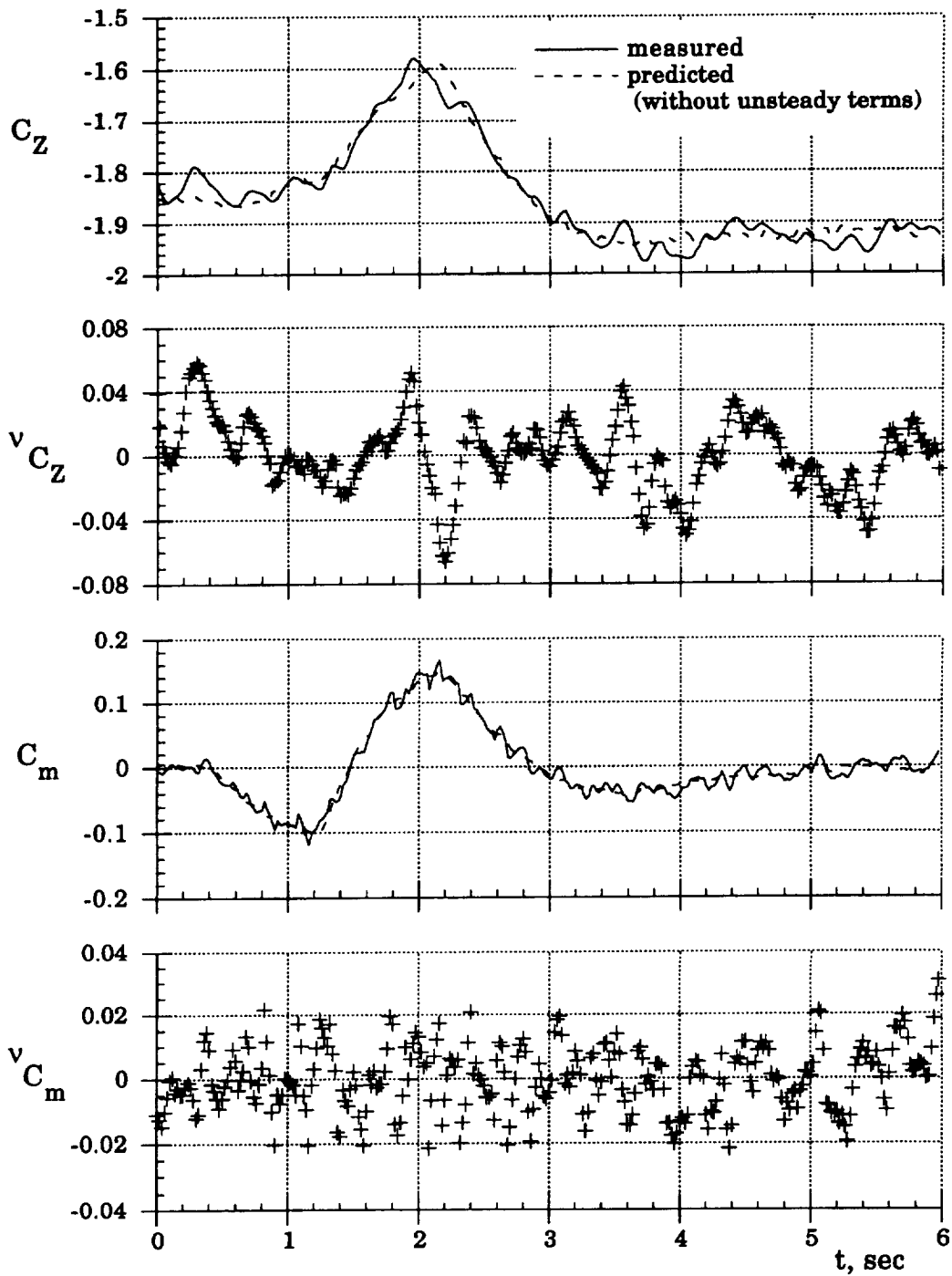


Figure 9. Time histories of measured and predicted aerodynamic coefficients and residuals. X-31A aircraft.

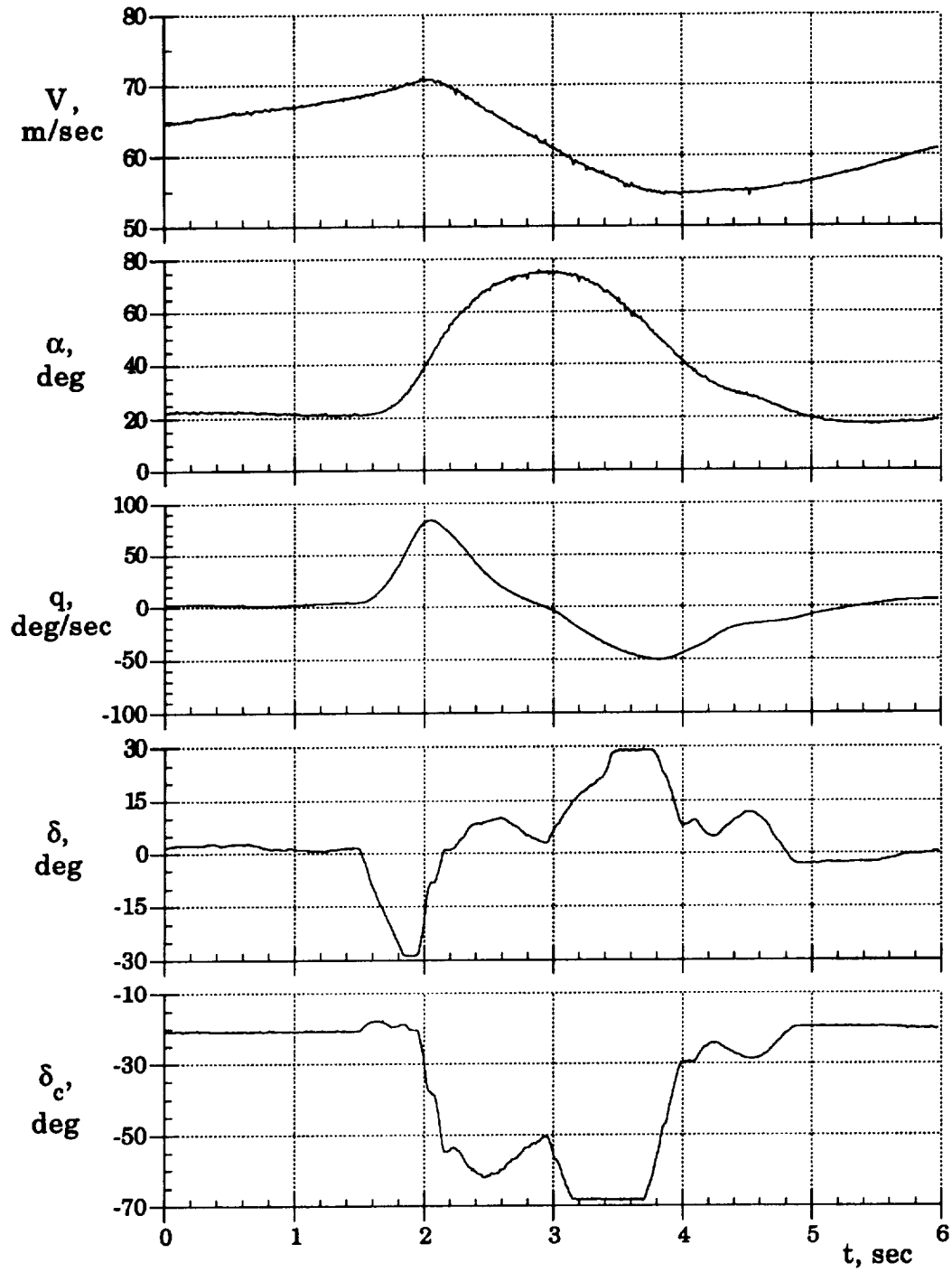


Figure 10. Time histories of measured longitudinal variables.
X-31 drop model.

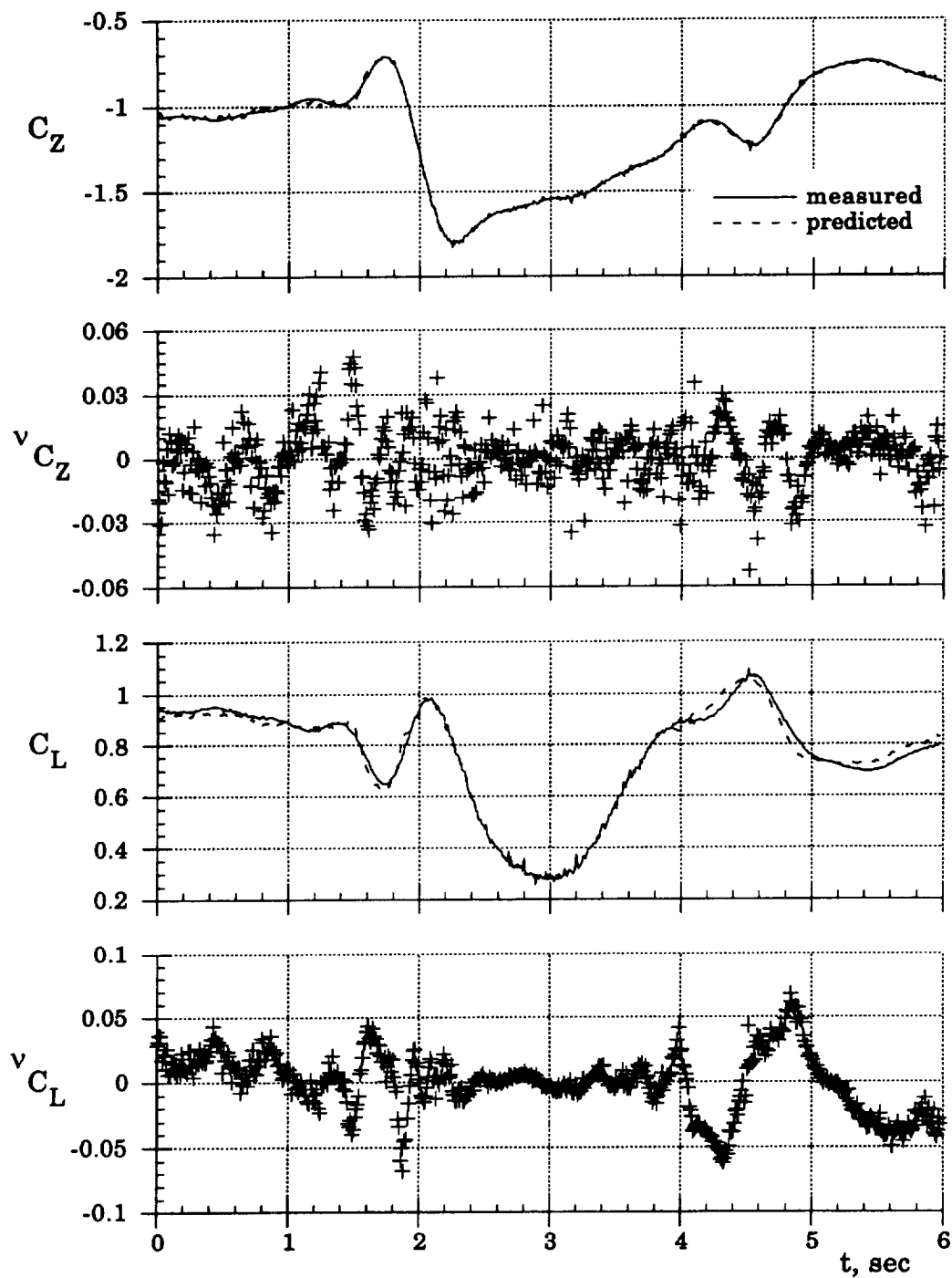


Figure 11. Time histories of measured and predicted aerodynamic coefficients and residuals. X-31 drop model.

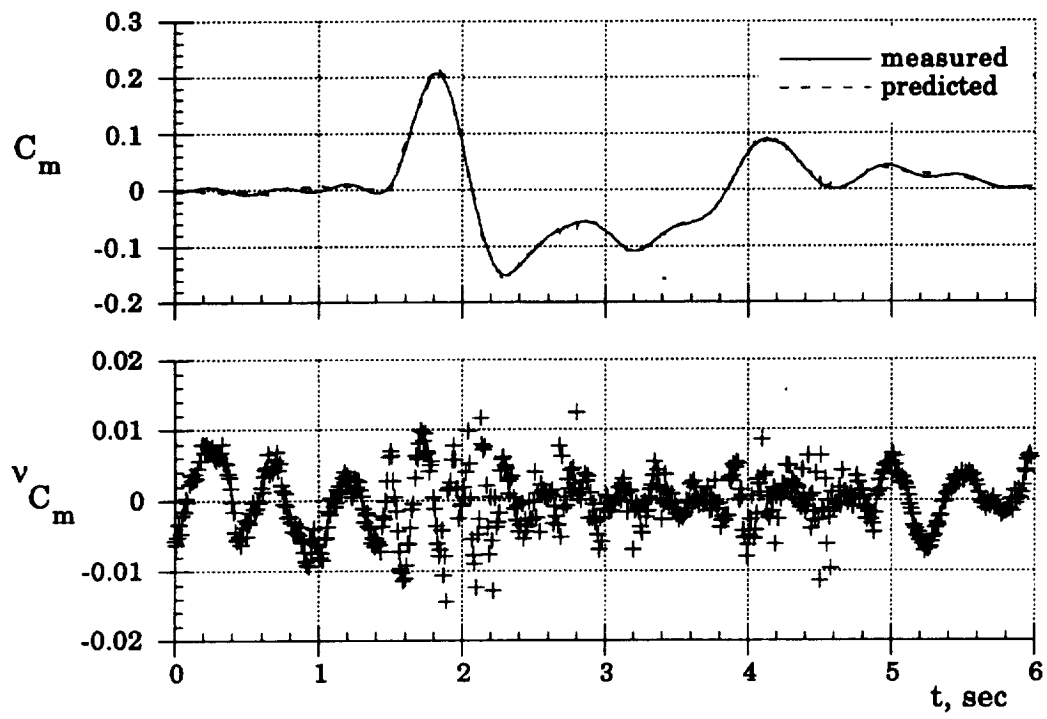


Figure 11. Concluded.

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13. ABSTRACT (Maximum 200 words) A nonlinear least squares algorithm for aircraft parameter estimation from flight data was developed. The postulated model for the analysis represented longitudinal, short period motion of an aircraft. The corresponding aerodynamic model equations included indicial functions (unsteady terms) and conventional stability and control derivatives. The indicial functions were modeled as simple exponential functions. The estimation procedure was applied in five examples. Four of the examples used simulated and flight data from small amplitude maneuvers for the F-18 HARV and X-31A aircraft. In the fifth example a rapid, large amplitude maneuver of the X-31 drop model was analyzed. From data analysis of small amplitude maneuvers it was found that the model with conventional stability and control derivatives was adequate. Also, parameter estimation from a rapid, large amplitude maneuver did not reveal any noticeable presence of unsteady aerodynamics.				
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